2501/103 2508/103
2502/103 2509/103
2503/103
ENGINEERING MATHEMATICS I
Oct./Nov. 2021
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN MECHANICAL ENGINEERING (PRODUCTION OPTION) (PLANT OPTION)
DIPLOMA IN AUTOMOTIVE ENGINEERING
DIPLOMA IN WELDING AND FABRICATION
DIPLOMA IN CONSTRUCTION PLANT ENGINEERING

MODULE I

ENGINEERING MATHEMATICS 1

3 hours

INSTRUCTIONS TO THE CANDIDATES

You should have the following for this examination:
- *Answer booklet;*
- *Mathematical tables/Non-programmable scientific calculator;*
- *Drawing instrument.*

This paper consists of ***EIGHT*** *questions.*
Answer any ***FIVE*** *questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Simply $\dfrac{\frac{1}{4}+2\frac{1}{7}\div\frac{2}{7}\,of\,3\frac{2}{3}}{\frac{3}{4}\times\left(2\frac{1}{7}-\frac{1}{6}\right)}$. (6 marks)

(b) Solve the equation:

$3^{2x+1}-10(3^{x})+3=0$ (6 marks)

(c) Solve the simultaneous equations:

$3\log_5 x-2\log_4 y=2$

$2\log_5 x+3\log_4 y=10$ (8 marks)

2. (a) Given the complex numbers $z_1=3+4j$, $z_2=5-6j$ and $z_3=4+7j$.

Determine $z=z_1-\dfrac{(z_2+z_3)}{z_2 z_3}$ in polar form. (10 marks)

(b) Simplify $\dfrac{(\cos 2\theta+j\sin 2\theta)^5}{(\cos 4\theta-j\sin 4\theta)^3}$ (3 marks)

(c) Given that $z=2+j$ is a root to the equation $z^3-6z^2+13z+w=0$ where w is a constant.

Determine the:

(i) value of w;

(ii) other roots. (7 marks)

3. (a) Determine the ratio of the constant terms in the binomial theorem expansion of

$\left(2x^2+\dfrac{1}{x}\right)^9$ and $\left(2x^3+\dfrac{3}{x^2}\right)^9$ (7 marks)

(b) Determine the number of ways in which a committee of 5 members can be formed from 10 men and 7 women to include:

(i) at least three men;

(ii) at most two men. (8 marks)

(c) Three quantities x, y and z are related by the equation $R=\dfrac{x^{\frac{1}{2}}y^{\frac{1}{3}}}{z^{\frac{1}{4}}}$. Use the binomial theorem to determine the approximate change in R. If x increases by 2% y decreases by 3% and z decreases by 5%. (5 marks)

4. (a) The sum of three numbers in a arithmetic progression is 27 and the sum of their squares is 293. Determine the:

(i) numbers;

(ii) sum of the first 10 terms of the series. (11 marks)

(b) Use geometric progression to convert the recurring decimal $0.4\dot{2}\dot{3}$ as a fraction. (9 marks)

5. (a) Prove the identities:

(i) $\dfrac{\cos 2\theta}{\cos\theta} - \dfrac{1-\tan^2\theta}{\sec\theta} = -2\tan\theta\sin\theta$

(ii) $\dfrac{\cos^2\theta - 7\sin 2\theta + 45\sin^2\theta}{\cos^2\theta - 9\sin\theta\cos\theta} = 1 - 5\tan\theta$ (9 marks)

(b) (i) Express $8\sin\theta + 5\cos\theta$ in the form $R\sin(\theta+\alpha)$ where α is an acute angle.

(ii) Hence solve the equation:

$8\sin\theta + 5\cos\theta = 7$, for $0° \le \theta \le 360°$. (11 marks)

6. (a) Prove the hyperbolic identities:

(i) $\cosh(x+y) = \cosh x\cosh y + \sinh x\sinh y$

(ii) $\dfrac{\cosh^2 x + 3\sinh 2x + 8\sinh^2 x}{\cosh^2 x + \sinh 2x} = 1 + 4\tanh x$ (9 marks)

(b) Determine the values of P and Q if $P\cosh x + Q\sinh x = 5e^x + 7e^{-x}$. (5 marks)

(c) Solve the equation:

$8\sinh 3x - 5\cosh 3x = 7$ (6 marks)

7. (a) Given the function $f(x) = \dfrac{4+3x}{9-5x}$ determine:

(i) $f^{-1}(0)$

(ii) $f^{-1}(1)$ (7 marks)

(b) Show that:

$\cos^{-1}\dfrac{3}{5} + \sin^{-1}\dfrac{5}{13} = \dfrac{63}{65}$ (6 marks)

(c) Determine the logarithmic form of $\coth^{-1} x$. (7 marks)

8. (a) Show that the equation of the parabola $4y^2 + 12x - 9 = 0$ is $\frac{3}{2(\cos\theta - 1)}$. (7 marks)

(b) Use De Moivre's theorem to express $\sin^5\theta$ in terms of sines of multiples of θ. (6 marks)

(c) Three currents I_1, I_2 and I_3 in amperes, flowing in a d.c network satisfy the equations:

$$3I_1 - I_2 + I_3 = 7$$
$$I_1 + 2I_2 + 3I_3 = 20$$
$$2I_1 + 4I_2 + I_3 = 20$$

Use elimination method to solve the equations. (7 marks)

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