2601/103
2602/103
2603/103
ENGINEERING MATHEMATICS I
June/July 2021
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical table/Scientific calculator;
Drawing instruments.
*This paper consists of **EIGHT** questions.*
*Answer **FIVE** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Simplify the expressions:

(i) $\dfrac{(1-x^3)^{-\frac{1}{2}} - x(1-x^3)^{\frac{1}{2}}}{(1-x^3)}$.

(ii) $\dfrac{\log 1024 - \frac{1}{2}\log 256 + 3\log 16}{\log 64 - \frac{1}{2}\log 16 + \log 4}$.

(7 marks)

(b) Solve the equations:

(i) $\log_2 4^{2x} = \log_3 27^{(x-1)}$;

(ii) $16\log_2 x + 4\log_4 x + 2\log_{16} x = 37$;

(iii) $\log_y 27 = 3 + \dfrac{1}{\log_8 y}$.

(13 marks)

2. (a) Given that the complex numbers $z_1 = 3 + 4j, z_2 = 6 - 7j$ and $z_3 = 1 + j$, express $z = z_1 + \dfrac{z_2 z_3}{z_2 + z_3}$ in the form $a + jb$. (8 marks)

(b) Use De Moivre's theorem to prove that $\sin 3A = 3\sin A - 4\sin^3 A$. (5 marks)

(c) Given that $z = 1 + j$ is a root of the equation $z^4 - 6z^3 + 15z^2 - 18z + 10 = 0$, determine the other roots. (7 marks)

3. (a) Given that $P\cosh 2x + Q\sin h\, 2x \equiv 4e^{2x} - 3e^{-2x}$, determine the values of P and Q. (7 marks)

(b) Prove the identities:

(i) $\cosh \dfrac{x}{2} = \sqrt{\dfrac{\cosh x + 1}{2}}$;

(ii) $\tan h\, \dfrac{x}{2} = \sqrt{\dfrac{\cosh x - 1}{\cosh x + 1}}$.

(7 marks)

(c) Solve the equation $4\cosh 3x - 5\sin h\, 3x = 3$ correct to 3 decimal places. (6 marks)

4. (a) Find the derivative of $y = \sin 2x$ from first principles. (6 marks)

(b) By using implicit differentiation, determine the equation of the normal to the curve $x^2 + y^2 + 3xy - 8x - 3y + 6 = 0$ at the point (1, 1). (6 marks)

(c) Determine the turning points of the function:

$f(x) = \dfrac{x^3}{3} + \dfrac{x^2}{2} - 6x + 7$ and state their nature. (8 marks)

5. (a) Determine the fourth term in the binomial expansion of $(x+3y)^{12}$ and find its value when $x=\frac{1}{2}$ and $y=\frac{1}{3}$. (5 marks)

(b) Determine the first four terms in the binomial expansion of $\left(9-\frac{x}{3}\right)^{\frac{1}{2}}$ and state its range of validity. (5 marks)

(c) Show that for small values:

(i) of x, $\sqrt{\dfrac{1-\frac{x}{4}}{1+\frac{x}{4}}}=1-\frac{x}{4}+\frac{x^2}{32}$ approximately;

(ii) By setting $x=\frac{1}{16}$, determine $\sqrt{\frac{63}{65}}$ approximately correct to 4 decimal places. (10 marks)

6. (a) Use factorization method to solve the equation $\frac{2}{(x-2)}+\frac{3}{(x-3)}=4$. (7 marks)

(b) The roots of the equation $ax^2+bx+c=0$ are $(\alpha-2)$ and $(\alpha+2)$ where a, b, c and α are constants. Prove that $b^2=4a(c+4a)$. (6 marks)

(c) The currents in a d.c circuit satisfy the simultaneous equations:

$$2I_1+I_2-3I_3=-5$$
$$I_1-2I_2+4I_3=9$$
$$3I_1+I_2+2I_3=7$$

Use the method of elimination to solve for I_1, I_2 and I_3. (7 marks)

7. (a) Solve $2\sin^2\theta+3\sin\theta=2$ for $0<\theta<360^\circ$. (3 marks)

(b) Prove the identities:

(i) $\frac{\cos A+\sin A}{\cos A-\sin A}=\sec 2A+\tan 2A$;

(ii) $\sin 2\phi+\sin\phi+\sin 3\phi=2\sin 2\phi(1+2\cos\phi)$;

(iii) $\frac{\cos 2A+\sin 2A-1}{\cos 2A-\sin 2A+1}=\tan A$.

(9 marks)

(c) Given that $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$. A is obtuse and B is acute. Determine:

(i) Sin (A + B);

(ii) tan (A- B).

(8 marks)

8\. (a) Given that $z = f(x, y) = \cos(ax + by)$, Show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = -(a^2 + b^2)z$ where a and b are constants.

(5 marks)

(b) Evaluate the integrals:

(i) $\int \frac{2x+4}{(x-1)(x-2)}\,dx$;

(ii) $\int \cos^2 \frac{\theta}{2}\,d\theta$;

(iii) $\int \frac{(e^{3x-1} - 4)}{2e^x}\,dx$.

(10 marks)

(c) Using calculus, determine the root mean square value of the voltage function $V = 50\sin(10t)$ volts in the interval $t = 0$ to $t = \pi$ correct to 2 decimal places.

(5 marks)

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