2601/103 2603/103
2602/103
ENGINEERING MATHEMATICS I
June/July 2022
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Drawing instruments;
Mathematical tables/ non-programmable Scientific calculator.
*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) The constant term in the expansion of $\left(ax+\frac{b}{x}\right)^{10}$ is 8064 where a and b are constants. Express a in terms of b. (8 marks)

(b) Expand $\frac{1+3x}{1-2x}$ as far as the term x^2 and state the values of x for which the expansion is valid. (6 marks)

(c) By expanding $(1+x)^{\frac{1}{4}}$ upto the term in x^3, approximate the value of $\sqrt[4]{50}$ correct to 3 decimal places. (6 marks)

2. (a) Find $\frac{dy}{dx}$, given that:

(i) $y=x^x$

(ii) $y=x^3\sin^2(4x)$

(iii) $x^2y+xy^2+y^3=0$

(7 marks)

(b) Given that $f(x,y)=x^3y^2+\sin(xy)+e^{xy}$. Show that $\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial^2 f}{\partial y\partial x}$. (8 marks)

(c) The volume V of a cone of radius r and height h is given by $V=\frac{1}{3}\pi r^2h$. If r is decreased by 1% and h is increased by 3%, find the percentage change in V by using partial differentiation. (5 marks)

3. (a) (i) By setting $t=\tan\left(\frac{x}{2}\right)$, find:

$$\int\frac{dx}{(5+3\cos x)}$$

(ii) Evaluate the integral

$$\int_3^5\frac{2x+3}{(x+1)^2(x-2)}dx$$

(14 marks)

(b) (i) Sketch the region bounded by the parabolas:

$y=x^2-x$ and $y=x-x^2$

(ii) By using integration, determine the area bounded in (b)(i) above. (6 marks)

$ax^n = anx^{n-1}$

$\sin ax = a\cos ax$

$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$ $\cos ax = -a\sin ax$

$e^{ax} = ae^{ax}$

$\ln ax = \frac{1}{}$

4. (a) (i) Given that $f(x)=\dfrac{x-2}{x+2}$.

Find $f^{-1}(x)$.

(ii) Convert $r=\sec\theta\operatorname{cosec}\theta$ to cartesian form.

(7 marks)

(b) The roots of the quadratic equation $2x^2+7x+3=0$ are α and β. Find an equation whose roots are α^2 and β^2 without solving the equation. (6 marks)

(c) Three currents I_1, I_2 and I_3 in amperes in a d.c network satisfy the equations

$7I_1 + 5I_2 = 25$

$5I_1 + 19I_2 - 4I_3 = 25$

$-4I_2 + 6I_3 = 50$

Use the method of substitution to solve the equations. (7 marks)

5. (a) Use exponential functions to prove that:

$\dfrac{2e^{2x}+2e^{-2x}}{2e^{2x}-2e^{-2x}}$ $\dfrac{(2e^{2x}+2e^{-2x})}{2e^{2x}+2e^{-2x}}$

(i) $\cosh(2x)=\cosh^2(x)+\sinh^2(x)$

(ii) $\sinh(x+y)=\sinh x\cosh y+\sinh y\cosh x$

(iii) $\coth^2 x-\cosh^2 x=1$ (7 marks)

$4e^{4x}+4e^{-4x}+4e^{-4x}+4e^{4x}$

$8e^{4x}+$

(b) (i) Show that $\coth^{-1}x=\dfrac{1}{2}\ln\left(\dfrac{x+1}{x-1}\right)$

(ii) Hence evaluate $\coth^{-1}(3)$ correct to 3 decimal places.

(7 marks)

(c) Solve the equation $3\operatorname{sech}^2 x+4\tanh x+1=0$, correct to 3 decimal places.

(6 marks)

$(a+b)$

6. (a) Simplify the expression

$a(a+b)+b(a+b)$

$a^2+ab+ab+b^2$

$$\frac{\log_8\left(\frac{1}{2}\right)+\log_4\left(\frac{1}{16}\right)}{\log_{\left(\frac{1}{2}\right)}(8)+\log_{\left(\frac{1}{16}\right)}4}$$

(6 marks)

$a^2+2ab+b^2$

$\cosh(2x)=\left(\dfrac{e^{x}-e^{-x}}{2}\right)\times 2$

$(2e^{2x}+2e^{-2x})(2e^{2x}+2e^{-2x})$

(b) Solve the equations:

(i) $\log_{10}(1+\sqrt{x})=\frac{1}{2}\log_{10}(9+\sqrt{16x})$

(ii) $\log_2(x^2y)=2$

$11+\frac{1}{2}\log_2 y=3\log_2 x$

$f(x) = \frac{(x+h)+(x+h)}{h}$

$P.R = V\frac{du}{dx} + U\frac{dv}{dx}$

(14 marks)

7. (a) Differentiate the function

$r = \sqrt{x^2+y^2}$

$f(x)=\frac{1}{2-x}$ from first principles. $\theta = \tan^{-1}\frac{y}{x}$ (6 marks)

(b) The normal to the curve $y=\frac{16}{x}-4\sqrt{x}$ at the point $(4,-4)$ intersects the y-axis at point P. Determine the co-ordinates of P. (5 marks)

(c) Locate the stationary points on the curve $f(x,y)=3x^2-y^3+6xy+4$ and determine their nature. (9 marks)

8. (a) Express $z=\frac{j}{1+j}$ in exponential form giving your answer in surd form.

(5 marks)

(b) Use De Moivre's theorem to show that $\cos 5A=16\cos^5 A-20\cos^3 A+5\cos A$.

$(r\angle\theta)^n = r^n\angle n\theta$

(7 marks)

(c) One root of the equation $2Z^3-5Z^2+aZ-5=0$ is $z=1-2j$. Determine the:

(i) value of the constant a

(ii) other two roots.

(8 marks)

$1+nx+\frac{n(n-1)x^2}{2!}+\frac{n(n-1)(n-2)x^3}{3!}$

$(a+x)^n = a+na^{n-1}x+\frac{n(n-1)a^{n-2}x^2}{2}+\frac{n(n-1)a^{n-2}x^3}{3!}$

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