2521/102 2602/103
2601/103 2603/103
ENGINEERING MATHEMATICS I
Oct./Nov. 2022
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
- *Answer booklet;*
- *Drawing instruments;*
- *Mathematical tables/Non-programmable scientific calculator.*

This paper consists ***EIGHT*** *questions.*
Answer any ***FIVE*** *questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given that $y = e^{3x}$. Find $\frac{dy}{dx}$ from first principles. (5 marks)

(b) A curve is parametrically defined by $x = 2\cos 3t$ and $y = 2\sin 3t$. Determine the equation of the normal to the curve at $t = \frac{\pi}{9}$. (7 marks)

(c) Locate the stationary points of the function $f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 12x - \frac{100}{3}$ and determine their nature. (8 marks)

2. (a) Simplify the expressions:

(i) $$\frac{(1+x)^{\frac{1}{2}} - \frac{1}{2}x(1+x)^{-\frac{1}{2}}}{(1+x)^{\frac{3}{2}}}$$

(ii) $$\frac{\log 125 - \frac{1}{2}\log 25}{\log 625 + \frac{1}{2}\log 25}$$ (6 marks)

(b) Solve the equations:

(i) $\log_{4x}\left(\frac{1}{2}\right) - \log_{\left(\frac{x}{2}\right)}\left(\frac{1}{2}\right) = \frac{3}{4}$

(ii) $2(3^{2x}) - 7(3^x) + 3 = 0$, correct to 2 decimal places. (14 marks)

3. (a) By using exponential definitions, prove that:

(i) $\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

(ii) $1 - \tanh^2 x = \operatorname{sec}h^2 x$

(iii) $2\sinh x \cosh x = \sinh 2x$. (8 marks)

(b) Show that $\operatorname{sec}h^{-1}x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$. Hence evaluate $\operatorname{sec}h^{-1}(0.2)$ correct to 3 decimal places. (6 marks)

(c) Solve the hyperbolic equation $6\cosh 2x - 5\sinh 2x = 4$ correct to 3 decimal places. (6 marks)

4. (a) Given that $z_1 = 2 - 3j$, $z_2 = 3 + 4j$ and $z_3 = 4 + j$ express Z in the form $a + jb$ if $\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}$ correct to 3 decimal places. (7 marks)

(b) Use De Moivre's theorem to show that $\cos 4A = 8\cos^4 A - 8\cos^2 A + 1$. (6 marks)

(c) Given that $z = 2 - j$ is a root of the equation $z^4 - 6z^3 + 15z^2 - 18z + 10 = 0$, determine the other roots. (7 marks)

5. (a) Show that the polar form of the Cartesian equation $x^2+xy+y^2-2x=0$ is given by $r=\dfrac{2\cos\theta}{1+\sin\theta\cos\theta}$. (4 marks)

(b) The roots of the quadratic equation $ax^2+bx+c=0$ differ by 1. Prove that $b^2=a(a+4c)$. (6 marks)

(c) Figure 1 shows a d.c network of resistors. By using elimination method, determine the values of the current I_1, I_2 and I_3 correct to 2 decimal places. (10 marks)

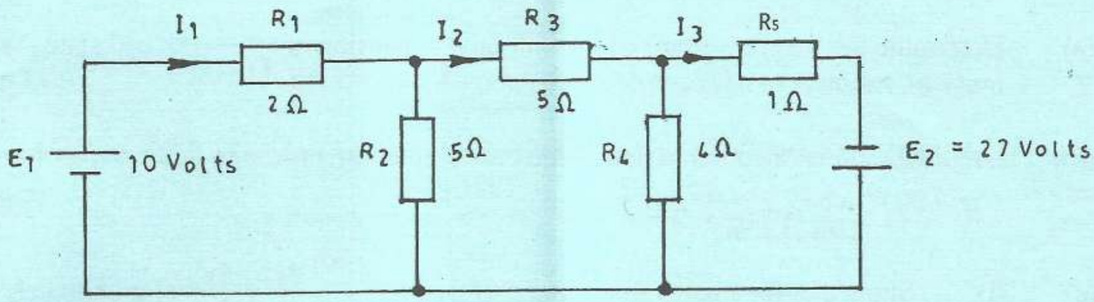

Fig. 1

6. (a) Given that $\sin\alpha=\frac{12}{13}$ and α is acute. By using suitable identities, determine in fractional form:

(i) $\sin 2\alpha$
(ii) $\cos 2\alpha$
(iii) $\tan 2\alpha$ (5 marks)

(b) (i) Given that A, B and C are angles of a triangle, prove that

$$\tan A+\tan B+\tan C=\tan A\tan B\tan C$$

(ii) Prove the identity

$$\tan\theta+\cot\theta=2\operatorname{cosec}2\theta$$ (10 marks)

(c) Solve the trigonometric equation $2\sin\beta+4\cos^2\beta=3$ for $0\leq\beta\leq 90^0$. (5 marks)

7. (a) The lateral surface area of a cone is calculated from the formula $s = \pi r\sqrt{r^2 + h^2}$, where r is the radius of the base and h is the vertical height. If r changes by 6% and h by 2%, use partial differentiation to determine the change in S when $r = 6$ cm and $h = 8$ cm correct to 2 decimal places. (6 marks)

(b) Evaluate the integral $\int_2^3 \frac{x}{(x-1)^2(x+2)}dx$ correct to 3 decimal places. (7 marks)

(c) Use integration to determine the area of the region enclosed by the line $y = 2x + 4$ and the curve $y = 2x^2$. (7 marks)

8. (a) Determine the first four terms of the binomial expansion of $\frac{1}{\left(6 - \frac{x}{2}\right)}$ and state the range of validity. (5 marks)

(b) Use the binomial theorem to determine the value of the fifth term in the expansion of $(3x + 5y)^{10}$ when $x = \frac{1}{3}$ and $y = \frac{1}{5}$. (5 marks)

(c) (i) Show that for small values of x, $\sqrt[3]{\frac{1 - \frac{x}{3}}{1 + \frac{x}{3}}} = 1 - \frac{2x}{9} + \frac{2x^2}{81}$ approximately.

(ii) By setting $x = \frac{2}{5}$ in (c)(i), determine the value of $\sqrt[3]{\frac{13}{17}}$ correct to 4 decimal places. (10 marks)

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