

1. (a) Solve the following equation for all values of  $\theta$  between  $0^\circ$  and  $360^\circ$ .  
 $2 \sin \theta - 3 \cos \theta = 2.$  (7 marks)
- (b) Solve for  $x$  in the following equations:  
 (i)  $\log_3(2x - 3) = -1;$  (3 marks)  
 (ii)  $3^{2x} = 4(3^x) + 3.$  (4 marks)
- (c) Find the minimum and maximum ordinates of the curve  $y = x^2(x+1)$  and classify them. (6 marks)
2. (a) (i) Expand  $\left(1 + \frac{2}{x}\right)^{\frac{1}{2}}$  upto the term in  $x^3$ . (6 marks)  
 (ii) Use the series above to determine  $(1.02)^{\frac{1}{2}}$  correct to 4 decimal places. (4 marks)
- (b) Given that  $x^2 + 3xy - y^2 = 3$ , find  $\frac{\delta^2 y}{\delta x^2}$  at the point (1,1). (6 marks)
- (c) The height  $S$  metres of a mass thrown vertically upwards at time  $t$  seconds is given by  $S = 90t - 17t^2$ . Determine how long the mass will take after being thrown to reach a height of 60 metres on the ascent and descent. (4 marks)
3. (a) Determine how many ways in which committees can be formed from a set of 5 governors and 7 senators if each committee contains 3 governors and 4 senators. (4 marks)
- (b) Evaluate  $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x \delta x.$  (6 marks)
- (c) Determine  $\int \frac{11-3x}{x^2+2x-3} \delta x.$  (10 marks)
4. (a) (i) Given that  $Z = \frac{1}{2+j3} + \frac{1}{1-j2}$ , express  $Z$  in the form  $a + jb$ . (5 marks)  
 (ii) Express  $Z = (2 - j7)$  in polar form. (4 marks)
- (b) If  $Z = 5x^4 + 2x^3y^2 - 3y$ , determine:  
 (i)  $\frac{\delta Z}{\delta x};$  (3 marks)  
 (ii)  $\frac{\delta Z}{\delta y}.$  (3 marks)

- (c) Show that if  $Z = \frac{x}{y} \ln y$ , then  $\frac{\delta Z}{\delta y} = \frac{x \delta^2 Z}{\delta y \delta x}$ . (5 marks)
5. (a) Determine by integration the area bounded by the three straight lines  $y = 4 - x$ ,  $y = 3x$  and  $3y = x$ . (7 marks)
- (b) The curve  $y = x^2 + 4$  is rotated one revolution about the x-axis between the limits  $x = 1$  and  $x = 4$ . Determine the volume of solid of revolution. (8 marks)
- (c) Find the derivative of the function  $y = 2x^2$  by first principles. (5 marks)
6. (a) Given that  $(a + b) + j(a - b) = 9 + j10$ , find the values of a and b. (4 marks)
- (b) Find the equation of the tangent to the curve  $y = x^2 - x - 2$  at the point (1, -2). (5 marks)
- (c) A rectangular sheet of metal having dimensions 20 cm by 12 cm has squares removed from each of the four corners and the sides bent upwards to form an open box. Determine the maximum possible volume of the box. (11 marks)
7. (a) The area A of a triangle is given by  $A = \frac{1}{2}ac \sin B$ , where B is the angle between sides a and c. If a is increasing at 0.4 units/s, c is decreasing at 0.8 units/s and B is increasing at 0.2 units/s, find the rate of change of the area of the triangle correct to 3 significant figures, when a is 3 units, c is 4 units and B is  $\frac{\pi}{6}$  radians. (10 marks)
- (b) Find the stationary points of the surface  $Z = x^3 - xy + y^3$  and distinguish between them. (10 marks)
8. (a) Simplify  $\frac{\cos 3x + j \sin 3x}{\cos x + j \sin x}$ . (3 marks)
- (b) Express  $Z = (-1 + j)$  in the form  $re^{j\theta}$ , where r is positive and  $-\pi < \theta < \pi$ . (4 marks)
- (c) Show that  $\frac{1 + \tan^2 B}{1 + \cot^2 B} = \tan^2 B$ . (6 marks)
- (d) The power P consumed in a resistor is given by  $P = \frac{V^2}{R}$  watts. Determine the approximate change in power when V increases by 5% and R decreases by 0.5% if the original values of V and R are 50 volts and 12.5 ohms respectively. (7 marks)