1. (a) Solve the equations:

(i) $\sqrt{x} = \frac{3+x}{\sqrt{12}}$

(ii) $3^{x^2} = 27^{x+1}$ correct to **two** decimal places.

(7 marks)

(b) Convert:

1 rad = 360°

(i) $3.21\angle 5.37$ radians to cartesian form;

(ii) $x^2 - y^2 = 16$ to polar form.

$\Rightarrow \sqrt{(x^2)^2(-y^2)^2} \Rightarrow$

(6 marks)

(c) The cost of 5 resistors, 4 capacitors and 1 diode is Ksh 340; the cost of 10 resistors, 9 capacitors and 4 diodes is Ksh 880; while the cost of 10 resistors, 13 capacitors and 15 diodes is Ksh 1920. Use elimination method to determine the cost of each component.

(7 marks)

2. (a) The word 'OPTICAL' is to be arranged so that the vowels always appear together. Determine the number of possible ways in which this can be done. (4 marks)

(b) (i) Use the binomial theorem to expand $\left(\frac{1-2x}{1+3x}\right)^{\frac{1}{3}}$ upto the term in x^2.

(ii) Hence, evaluate $\left(\frac{0.98}{1.03}\right)^{\frac{1}{3}}$ correct to **four** decimal places.

(10 marks)

(c) The second moment of area of a rectangle through its centroid is given by $I_G = \frac{bl^3}{12}$, where b is the width and l is the length. Use the binomial theorem to determine the approximate change in second moment of area if b increases by 2.5% and l decreases by 1.5%. (6 marks)

3. (a) Given the complex numbers $Z_1 = 6j$ and $Z_2 = 3 + j$, determine $\frac{Z_1}{Z_2}$, expressing the answer in exponential form. (5 marks)

(b) Solve the equation $Z^3 - 2 + j = 0$, giving the answer in the form a + jb.

(9 marks)

$Z = [2 - j]^{1/3} \Rightarrow$

(c) Use Demoivre's theorem to show that:

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

$Z = (r[1 - \cos\theta])^n$

(6 marks)

4. (a) Solve the equation $\log(x^2+6) - \log x = \log 5$. (3 marks)

(b) Find the value of $\tan\theta$, given that $\text{Sin}(\theta+45^\circ) = 3\cos(\theta+45^\circ)$. (4 marks)

(c) Solve the equation $4\,\text{Cos}\,2\theta - 2\,\text{Sin}\,\theta + 2 = 0$ for $0^\circ \le \theta \le 360^\circ$. (6 marks)

(d) The longest side of a right angled triangle is $(x+9)$ cm. If the lengths of the other remaining sides are $(x+5)$ cm and $(2x+6)$ cm, determine the:

(i) dimensions of the triangle;
(ii) area of the triangle.

(7 marks)

$-6-3=-9$

5. (a) Find the inverse of the function $f(x) = \dfrac{3x+2}{x-2}$. $\begin{bmatrix} 3 & 2 \\ 1 & -2 \end{bmatrix} = \frac{1}{-9}$ (3 marks)

(b) Given that $Ae^{x} - Be^{-x} = 8\cosh x - 2\sinh x$, find the values of A and B. (3 marks)

(c) Prove the hyperbolic identities:

(i) $\tanh(\theta-\phi) = \dfrac{\tanh\theta - \tanh\phi}{1-\tanh\theta\tanh\phi}$

(ii) $\dfrac{\sinh^2\theta + \cosh^2\theta - 1}{4\,\text{Cosh}^2\theta\coth^2\theta} = \dfrac{1}{2}\tanh^4\theta$

(7 marks)

(d) Solve the equation $4\cosh 2x = 4 + 2\sinh 2x$ giving the answer correct to three decimal places. (7 marks)

6. (a) Given $y = \sin(2x+3)$, find $\dfrac{dy}{dx}$ from first principles. (5 marks)

(b) Given that $y = \ln\left(\dfrac{1-x^2}{1+x^2}\right)$, show that $\dfrac{dy}{dx} = \dfrac{-4x}{1-x^4}$. (6 marks)

(c) The power developed in a resistor R by a battery of emf E and internal resistance r is given by $P = \dfrac{E^2R}{(R+r)^2}$. $\to R^2P + r$

(i) find $\dfrac{dp}{dR}$;

(ii) show that the power is maximum when $R = r$.

(9 marks)

7. (a) If $Z = \frac{x}{y} \ln y$, show that $\left(\frac{1}{y\ln y} - \frac{1}{y}\right)\frac{\partial z}{\partial x} = \frac{\partial^2 z}{\partial x \partial y}$. (5 marks)

(b) Given that the volume of a cone is $V = \frac{1}{3}\pi r^2 h$, use partial differentiation to determine the approximate change in volume if the radius increases from 5 cm to 6 cm, and the height decreases from 4 cm to 3.5 cm. (6 marks)

(c) Locate the stationary points of the function $Z = 2x^2 - 3y^2 + 8xy - 4x + 6y + 6$ and determine their nature. (9 marks)

8. (a) Evaluate the integrals:

(i) $\int (3x-5)^4 \, dx$

(ii) $\int \text{Sin}^2 3x \, dx$

(iii) $\int_6^7 \frac{18 + 21x - x^2}{(x-5)(x+2)^2} dx$

(12 marks)

(b) Figure 1 shows a sketch of the graph of the function $y = e^x$.

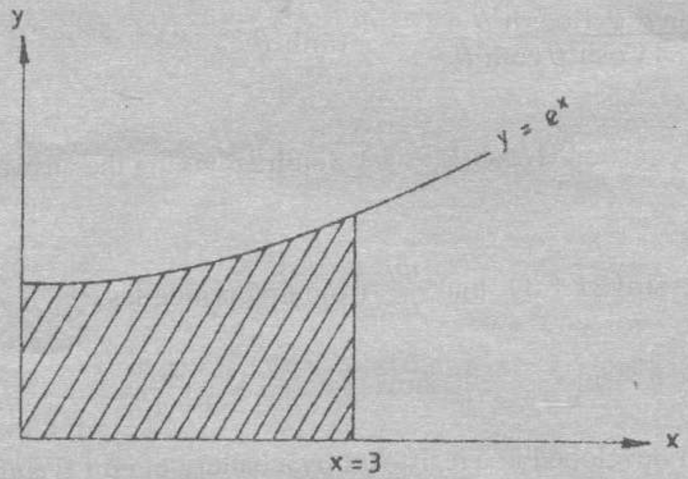

Fig. 1

Determine the:

(i) area enclosed by the curve, the x-axis, the y-axis and the ordinates $x = 3$.

(ii) centroid of the area in (i) above.

(8 marks)

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