2521/201, 2602/203
2601/203, 2603/203
ENGINEERING MATHEMATICS II
June/July 2023
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRICAL AND ELECTRONIC ENGINEERING
(POWER OPTION)
(TELECOMMUNICATION OPTION)
(INSTRUMENTATION OPTION)

MODULE II

ENGINEERING MATHEMATICS II

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator;
Abridged tables of Laplace transform standard normal table.
*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** questions in the answer booklet provided.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 7 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1\. (a) Given the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

(i) Show that $A^3 + A^2 - 21A - 45I = 0$

(ii) hence determine A^{-1}

(8 marks)

(b) Use the inverse matrix method to solve the following simultaneous equations:

$$4R + 2S + T = 19$$
$$3R + 6S + 2T = 34$$
$$2R + 3S + 7T = 48$$

(12 marks)

2\. (a) The marks obtained by Diploma in Electrical Engineering students in a mathematics test are shown in table 1.

Table 1

Marks	0 - 6	7 - 13	14 - 20	21 - 27	28 - 34	35 - 41	42 - 48	49 - 55
No. of students	2	3	4	5	6	4	3	3

Determine the:

(i) mean mark;

(ii) variance;

(iii) median.

(11 marks)

(b) Two regression equations of variables of x and y are:

$x = 15.14 - 0.69y$

$y = 10.84 - 0.63x$

Determine:

(i) mean of x;

(ii) mean of y;

(iii) coefficient of correlation between x and y.

(9 marks)

3. (a) Determine the Laplace transform of $\sin 4t$ from first principles. (5 marks)

(b) User Laplace transforms to solve the differential equation:

$$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 10y = \cosh 3t$$

given that when $t = 0, y = 1$ and $\frac{dy}{dt} = 4$. (15 marks)

4. (a) Given the coplanar vectors $\underline{A} = 2\underline{i} - \underline{j} + \underline{k}$, $\underline{B} = \underline{i} + 2\underline{j} - 3\underline{k}$ and $\underline{C} = 3\underline{i} + Z\underline{j} - 5\underline{k}$.

Determine the value of Z.

(5 marks)

(b) Determine the directional derivative of the scalar field $\phi = xy^2z + 2x^2y + 3xz^3$ at the point (1,1,1) in the direction of the vector $\underline{A} = \underline{i} + \underline{j} - \underline{k}$. (7 marks)

(c) A rigid body is rotating at the rate of 5 revolutions per second about an axis through the origin whose direction ratios are $(2, -1, 3)$.
Determine the magnitude of the velocity of the body at the point (2, 4, 3).

(8 marks)

5. (a) A random variable x has a probability density function as shown in table 2.

Table 2

x	0	1	2	3	4	5	6	7	8
$p(x)$	a	3a	5a	7a	9a	11a	13a	15a	17a

(i) Determine the:

(I) value of a;

(II) $P(x < 4)$.

(ii) Determine the value of a such that $P(x \leq 3) > 0.5$.

(7 marks)

(b) A continuos variable x has probability density function $f(x)$ given by:

$$f(x) = \begin{cases} kx(4x+5) & 0 \leq x \leq 2 \\ 0, & elsewhere \\ \text{where } k \text{ is a constant} \end{cases}$$

Determine the:

(i) value of k;

(ii) mean;

(iii) standard deviation.

(13 marks)

6. (a) Solve the differential equation:

$$(8y - x^2y)\frac{dy}{dx} + (x - xy^2) = 0$$

(8 marks)

(b) Use the method of undetermined coefficient to solve the differential equation:

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = e^{-3t},$$

given that when $t = 0, x = 2$ and $\frac{dx}{dt} = \frac{-7}{2}$

(12 marks)

7. (a) Determine the first three non-zero terms of the Maclaurin's series expansion of $f(x) = \tan \lambda x$ where λ is a constant. (9 marks)

(b) (i) Use Taylor's theorem to expand $f(x) = 3x^4 + 6x + 5$ in powers of $x - 2$;

(ii) Hence evaluate the integral $\int_0^1 \frac{3x^4 + 6x + 5}{(x-2)^4} dx$, (11 marks)

8. (a) Given that $z = 5\sin\left(\frac{y}{x}\right)$

Show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$

(6 marks)

(b) The deflection y at the edge is given by $y = \frac{kwd^4}{t^3}$, where w is the total load, d the diameter of the plate, t the thickness of the plate and k is a constant. Use partial differentiation to determine the percentage change in y if w increases by 4%, d is decreased by 3% and t is increased by 3%. (6 marks)

(c) Locate the stationary points of the function $z = x^2 + 3y^2 + 4xy - 20x - 32y + 20$ and determine their nature. (8 marks)