1. (a) Give the matrices

$$A = \begin{pmatrix} 1 & -3 & 0 \\ 1 & 2 & -1 \\ -2 & 4 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 4 & 3 \\ -5 & -3 & 0 \\ 1 & 2 & -2 \end{pmatrix}$$

Find AB - A. **(4 marks)**

(b) Solve the equation

$$\begin{vmatrix} x & 3 & 2 \\ 1 & 1 & x \\ x & 1 & 2 \end{vmatrix} = 28$$

(4 marks)

(c) Currents i_1, i_2, and i_3 in an electric circuit satisfy the equations,

$$\begin{aligned} i_1 + i_2 + i_3 &= 0 \\ 10i_1 - 8i_2 &= 14 \\ 8i_1 - 3i_3 &= 39 \end{aligned}$$

Determine, using Cramer's rule the values of the three currents.

(12 marks)

2. (a) If $z = x^3 + y^2 + 2xy$, show that

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$$

(5 marks)

(b) The radius of a right circular cylinder is increasing at the rate of 3 cm/s and the height is decreasing at the rate of 2 cm/s.
Find using partial differentiation the rate at which the volume is changing at the instant when the radius is 8 cm and the height is 12 cm.

(6 marks)

(c) Determine the stationary points of the function:

$f(x, y) = y^2 + 2x^2 - 2xy + 14y + 8$

and classify them.

(9 marks)

3. (a) Given the vectors,

$\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{b} = -3\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$,

Find:

(i) $\underset{\sim}{a} \cdot \underset{\sim}{b}$

(ii) $\underset{\sim}{a} \times \underset{\sim}{b}$

(iii) the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$

(9 marks)

(b) Given that

$\underset{\sim}{v} = y^2z\underset{\sim}{i} - 3x^2z\underset{\sim}{j} + 2xy^3\underset{\sim}{k}$

and

$\Phi = 3xy - 4yz + x^3z$,

Determine at the point (1, -1, 2),

(i) Grad **Φ**

(ii) Curl $\underset{\sim}{V}$

(6 marks)

(c) Find a unit vector that is normal to the vectors $\underset{\sim}{P} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{q} = \underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$

(5 marks)

4. (a) Use Taylor's theorem to determine the power series for $\cos(\frac{\pi}{3} + h)$ as far as the term in h^4, and hence determine the value of Cos 63° correct to five decimal places.

(8 marks)

(b) Use Maclaurin's theorem to expand the function $f(x) = \sin^2 x$ in ascending powers of x upto the term in x^6, and hence evaluate,

$\int_1^2 \frac{\sin^2 x}{x^3} dx$, correct to five decimal places.

(12 marks)

5. (a) Solve the differential equation $\frac{dy}{dx} + \frac{2}{x}y = x^2$, given that when $x = 1, y = 2$. (7 marks)

(b) Use the method of undetermined coefficients to solve the differential equation,

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 12x = e^{-4t}$$

given that when $t = 0, x = 1$ and $\frac{dx}{dt} = 3$.

(13 marks)

6. (a) Find the ;

(i) Laplace transform of $t^2\cos t$ (5 marks)

(ii) Inverse laplace transform of

$$F(s) = \frac{s^2 + 2s - 3}{s(s-3)(s+2)}$$ (6 marks)

(b) Use Laplace transforms to solve the differential equation

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 10x = e^{2t}$$

given that when $t = 0, x = 0$ and $\frac{dx}{dt} = 1$ (9 marks)

7. (a) 120 students pursuing a course in electrical engineering were examined and their results summarized as shown in table 1.

Table 1

Marks obtained	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Number of students	7	12	14	28	25	14	12	8

Using an assumed mean of 55, determine the:

(i) mean mark;
(ii) standard deviation;
(iii) Pearson's coefficient of skewness.

(13 marks)

(b) Table 2 shows the percentage marks obtained by ten students in mathematics and physics.

Table 2

Mark in mathematics	75	38	96	27	74	85	90	63	66	42
Marks in physics	85	51	92	60	64	68	88	62	65	48

(i) Determine the product moment correlation coefficient.

(ii) Hence, comment on the result.

(7 marks)

8. (a) The mean life span of 1200 electric bulbs is 15 hours and a standard deviation of 3 hours. Assuming the life span is normally distributed, determine the number of bulbs with a life span of:

(i) more than 18 hours;

(ii) between 12 hours and 21 hours.

(9 marks)

(b) A continuous random variable X has a probability density function f(x) defined by:

$$f(x)=\begin{cases} ke^{-\frac{1}{3}x} & , \quad \text{for } x \geq 0 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

where k is a constant.

Find the:

(i) value of k;
(ii) mean;
(iii) standard deviation of the distribution.

(11 marks)

TABLE OF LAPLACE TRANSFORMS

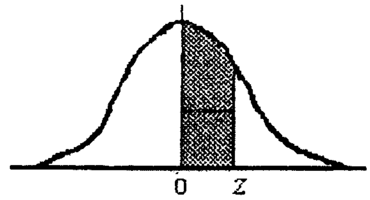
	FUNCTION $F(t)$	TRANSFORM $\int_0^{\infty} e^{-st} F(t)\,dt$
1.	1	$1/s$
2.	e^{at}	$1/(s-a)$
3.	$\sin at$	$a/(s^2+a^2)$
4.	$\cos at$	$s/(s^2+a^2)$
5.	t	$1/s^2$
6.	t^n (n a +ve integer)	$n!/s^{n+1}$
7.	$\sinh at$	$a/(s^2-a^2)$
8.	$\cosh at$	$s/s^2-a^2)$
9.	$t \sin at$	$2as/(s^2+a^2)^2$
10.	$t \cos at$	$(s^2-a^2)/(s^2+a^2)^2$
11.	$e^{-at}t^n$	$n!/(s+a)^{n+1}$
12.	$e^{-at}\cos \omega t$	$(s+a)/[(s+a)^2+\omega^2]$
13.	$e^{-at}\sin \omega t$	$\omega/[(s+a)^2+\omega^2]$
14.	$e^{-at}\cosh \omega t$	$(s+a)/[(s+a)^2-\omega^2]$
15.	$e^{-at}\sinh \omega t$	$\omega/[(s+a)^2-\omega^2]$

Some Theorems used in Laplace Transforms.

1. If $f(s) = L\{F(t)\}$, then $f(s+a) = L\{e^{-at} F(t)\}$
2. $L\{dx/dt\} = sL\{x\} - x(0)$ (b) $L\{d^2x/dt^2\} = s^2L\{x\} - sx(0) - x'(0)$

NORMAL CURVE

AREAS
under the
STANDARD
NORMAL CURVE
from 0 to z



z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.201	.2051	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

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